B.A/B.Sc 6th Semester (Honours) Special Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH6DSE31
(Mathematical Modelling)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5=30

(a) Investigate the stability of the steady state of the logistic difference equation $x_{n+1} = rx_n(1 - x_n)$.

(b) Derive the steady state difference equations for the queueing model $(M / M / 1) : (N / FCFS / \infty)$.

(c) Discuss Gompertz population model.

(d) Discuss biotic and abiotic factors of an ecosystem.

(e) Discuss different states of a queuing system.

(f) A population satisfies the growth equation $x_{n+2} - 2x_{n+1} + 2x_n = 0$. Find the population in n-th generation. Also, find the steady state.

(g) Explain the following:

(i) Poisson axioms of arrivals of a queueing system

(ii) Service discipline of a queueing system

(h) Discuss least squares estimator.

2. Answer any three questions: 3×10=30

(a) If the arrival process in a queueing system follows the Poisson distribution, then show that the associated random variable defined as inter-arrival time follows the exponential distribution.

(b) What is mutualism? Classify it and explain.

(c) Define a cooperative system with an example. Prove that the orbit of a system that is cooperative either converges to equilibrium or diverges to infinity.

(d) Discuss the linearization and stability of two species model equations.

(e) For the prey-predator system

$$\frac{dx}{dt} = x(1 - x - y)$$
$$\frac{dy}{dt} = \beta(x - \alpha)y$$

Investigate the nature of equilibrium points of the system.

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Subject: Mathematics

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(Industrial Mathematics)

Time: 3 Hours | Full Marks: 60

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Candidates are required to write their answers in their own words as far as practicable.

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1. Answer any six questions: **6×5=30**

(a) What do you mean by Hounsfield unit of a tissue? What is the typical clinical range of a CT scan between air and bone?

(b) Define Radon transform of a function with arguments t and θ. Explain it with an example.

(c) Define the unfiltered back projection for a function with arguments t and θ.

(d) Why is the graph of Radon transform called sinogram? Show that $l_{t,\theta} = l_{-t,\theta+\pi}$ for all t and θ.

(e) Draw the graph of the intensity of transmitted x-ray as function of time using Beer's law. Interpret the graph physically.

(f) Write and explain the algorithm of CT scan.

(g) Stating mathematical steps, briefly explain how the data can be derived from a century old mummy using CT scan keeping it intact.

(h) Evaluate the integrals:

(i) $\int_0^{2\pi} e^{i\theta} d\theta$ (ii) $\int_0^{\infty} e^{-\mu x} e^{-i\omega x}\, dx$

2. Answer any three questions: **3×10=30**

(a) Explain briefly why Beer's law is a plausible model for X-ray attenuation.

(b) Show that rotation ω can be defined as $\begin{pmatrix}\omega_x\\ \omega_y\\ \omega_z\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix} \times \begin{pmatrix}u_x\\ u_y\\ u_z\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\partial_y u_z - \partial_z u_y\\ \partial_z u_x - \partial_x u_z\\ \partial_x u_y - \partial_y u_x\end{pmatrix}$.

Hence define tilt. What are its physical significance?

(c) Show that the functions $F_1(t) = e^{(\alpha+i\omega)t}$ and $F_2(t) = e^{(\alpha-i\omega)t}$, where α and ω are real constants, satisfy the linear differential equation as follows:

$$y'' - 2\,\alpha y' + (\alpha^2 + \omega^2)y = 0.$$

Using Euler's formula, show that the functions $y_1 = e^{\alpha t}cos(\omega t)$ and $y_2 = e^{\alpha t}sin(\omega t)$ also satisfy the above differential equation.

(d) What is back projection? Give an example of it.

(e) For a given function *f*, show that $\mathcal{R}f(t,\theta) = \mathcal{R}f(-t,\theta+\pi)$, where $\mathcal{R}$ stands for Radon transformation.

B.A/B.Sc 6th Semester (Honours) Special Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH6 DSE33
(Group Theory-II)

Time: 3 Hours — Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: **6×5=30**

(a) Let $G = A \times A$ where A is a cyclic group of order p(prime). How many automorphisms does G have?

(b) Prove that any abelian group of order 2310 is cyclic.

(c) Show that $Aut(\mathbb{Z}\times\mathbb{Z})\approx$ GL (2, $\mathbb{Z}$), the group of 2 ×2 non-singular matrices over $\mathbb{Z}$.

(d) Prove that finite abelian group is isomorphic to product of p-groups.

(e) Let G be a group of order 12. Show that either 3-Sylow subgroup is normal or $G\approx A_4$.

(f) Let G be a finite group of prime order. Suppose that G acts on a set X and x is in X such that $ax=x$ for some a in G other than e. Then prove that $bx=x$ for all b in G.

(g) If each Sylow subgroup of G is normal then show that G is the direct product of its Sylow subgroups.

(h) Prove that any group of order p^2 where p is a prime is isomorphic to either $\mathbb{Z}_{p^2}$ or $\mathbb{Z}_p\times\mathbb{Z}_p$, where $\mathbb{Z}_p$ is integer modulo p.

2. Answer any three questions: **3×10=30**

(a) (i) If for a positive integer α, $o(G) = p^{\alpha}$, p is a prime number and also if $N\neq\{e\}$ is a normal subgroup of G, prove that $N\cap Z \neq \{ e\}$, where Z is the centre of G.

(ii) Prove that group of order 28 has a normal subgroup of order 7.

(b) (i) Let G be any group and A be non-empty subset of G. Then prove that any homomorphism from G into Sym(A), the symmetric group of A defines an action of G on A.

(ii) Let G and H be finite groups of order m and n respectively. Suppose that $\gcd(m,n)=1$. Then prove that $\mathrm{Aut}G\times\mathrm{Aut}H\approx\mathrm{Aut}(G\times H)$.

(c) (i) Give an example of an infinite 7-group. Justify your answer.

(ii) Show that there is no simple group G of order 216.

(d) (i) Determine all the conjugacy classes in S_4 and write down the classes of S_4.

(ii) Prove that the centre of S_n is trivial for $n>2$.

(e) (i) Let X be the set of all complex number with unit modulus and G be an additive group of reals numbers. Define $F: G\times X\to X$ by $F(a,x)=e^{ia}x$ for all a in G and x in X. Then prove that F is an action of G on X. Is F effective?

(ii) Let a group G act on itself by left translation. Prove that the action is effective and deduce the Cayley's theorem.