B.A/B.Sc. 1st Semester (General) Examination, 2020 (CBCS)
Subject: Mathematics
Paper: BMG1CC1A/MATH-GE1 (Differential Calculus)

Time: 3 Hours   Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

(a) (i) Show that $\lim_{x\to 0} cos\frac{1}{x}$ does not exist. [3]

(ii) Use Mean Value Theorem to show that $|\sin b - \sin a| \leq |b-a|$, for all real numbers a and b. [2]

(b) The function $f(x) = \begin{cases} e^x & if\ x \leq 1 \\ mx+b & if\ x > 1 \end{cases}$ is continuous and differentiable at $x = 1$. Find the values of the constants m and b. [5]

(c) State and prove Leibnitz's theorem for successive differentiation. [1+4]

(d) Trace the curve, $a^2x^2 = y^3(2a-y)$. [5]

(e) Determine the position and nature of the double points on the curve $y(y-6) = x^2(x-2)^3 - 9$. [5]

(f) (i) Does the function $f(x) = \begin{cases} x & \text{for } 0 \leq x < 1 \\ 0 & \text{for } x = 1 \end{cases}$ has a maximum on the interval $0 \leq x \leq 1$? [2]

(ii) Find the angle of intersection of the curves $x^2 - y^2 = a^2$ and $x^2 + y^2 = a^2\sqrt{2}$. [3]

(g) (i) Find the radius of curvature of the parabola $y^2 = 4x$ at the vertex (0, 0). [2]

(ii) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. [3]

(h) If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!}f''(\theta h), 0 < \theta < 1$ then find θ, when $h = 1$ and $f(x) = (1-x)^{5/2}$. [5]

2. Answer any three questions: $10\times3 = 30$

(a) (i) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{3}{(x+y+z)^3}$. [5]

(ii) If $V = \sin^{-1}\frac{x^2+y^2}{x+y}$ then prove that $xV_x + yV_y = \tan V$. [5]

(b) (i) Find the condition that the conics $ax^2 + by^2 = 1$ and $a_1x^2 + b_1x^2 = 1$ shall cut orthogonally. [5]

(ii) Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a and b are connected by the relation $ab = c^2$. [5]

(c) (i) Find the asymptotes of the curve $(x^2 - y^2)^2 - 4x^2 + x = 0$. [5]

(ii) Show that in any curve, $\rho = \left\{\left(\frac{dx}{d\psi}\right)^2 + \left(\frac{dy}{d\psi}\right)^2\right\}^{\frac{1}{2}}$. [5]

(d) (i) Find Maclaurin series for $x\sin(2x)$. [5]

(ii) Examine whether $x^{\frac{1}{x}}$ possesses a maximum or a minimum and determine the same. [5]

(e) (i) State Rolle's Theorem. Find all numbers c in the interval (0, 2) that satisfy the conclusion to this theorem for $f(x) = x^3 - 3x^2 + 2x + 5$. [2+4]

(ii) Find the Cauchy's reminder after n terms in the expansion of $\log(1 + x)$ in powers of x. [4]