

BCA (Honours) 1st Semester Examination, 2020

Subject: Mathematics-I

Paper: BCA-103

Time: 3 Hours

Full Marks: 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words

as far as practicable.

1. Answer any FIVE questions:

10x5=50

- a) Solve the equation $x^3 - 6x - 9 = 0$ by Cardan's method
- b) Solve by Cramer's rule: $x + 2y + 3z = -5$, $3x + y - 3z = 4$, $-3x + 4y + 7z = -7$
- c) Find the equation of parabola whose focus is $(2, 3)$ and the directrix is $4x - 3y + 1 = 0$.
Hence find the co-ordinates of vertex of the parabola.
- d) If pair of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ is such that each pair bisects the angles between the other pair, prove that $pq+1=0$.
- e) Find the locus of the middle point of the conic $\frac{l}{r} = 1 + e \cos\theta$.
- f) Define vector cross product. If the vectors $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$, $\vec{c} + \vec{d}$ are coplanar, then show that the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar.
- g) Discuss the nature of the conic represented by $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$ and reduce it to standard form.

2. Answer any SIX questions:

6x5=30

- a) If $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$, find the value of $3A$. Find B, if $B^{-1} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.
- b) Find whether or not the relations R_1 and R_2 in the set $A = \{1, 2, 3, 4\}$ are reflexive, symmetric, anti-symmetric, transitive (i) $R_1 = \{(1,1), (1,2)\}$ (ii) $R_2 = \{(1,1), (2,2), (4,4)\}$.

c) If a, b, c are roots of the equation $x^3 + 6x^2 + 12x - 19 = 0$, find the equation whose roots are $a+b, b+c, c+a$.

d) Prove, without expanding the determinant that $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$

e) Prove that finite integral domain is a field.

f) Find a complex number z such that $e^z = i$.

g) Prove that the product of a matrix and its transpose is a symmetric matrix.

h) Define mapping. If $A = \{p: -15 \leq p \leq 15\}$ and $B = \{q: 0 \leq q \leq 30\}$ find $A \cup B$ and $A - B$.